Welcome to 

which brings to a close the centennial year of quantum mechanics, IYQ.

May we continue to reap the benefits!

This is the 22nd consecutive winter physics meeting in the current series — a continuation of the "Coral Gables Conferences" that began in January 1964.

This year the meeting is once again almost entirely in-person. The hourly talk schedule is available online, along with a list of speakers and their affiliations. That schedule, and links to most abstracts, can be found on the conference website.

I hope your participation in Miami 2025 is enjoyable as well as productive and beneficial for your research.

Through your attendance, you join a long list of distinguished physicists who have participated in these meetings during the last 60+ years, including Barish, Dirac, Englert, Feynman, Gell-Mann, Glashow, Nambu, Reines, Salam, Schwinger, 't Hooft, Veltman, Weinberg, Wigner (altogether, about forty Nobel Laureates), and Zweig, to name a few that I had the pleasure to see in action.

Sic transit gloria mundi.

Bill Bardeen (1941-2025)

Luca Mezincescu (1946-2025)

Peter Minkowski (1941-2025)

Kelly Stelle (1948-2025)

C. N. Yang (1922-2025)

whose insightful and original work on quantum field theory, including gravity and supergravity, inevitably led to considerations of high-energy behavior, divergences, and renormalization.

These subjects have been discussed frequently at these meetings, from their very beginnings.

In particular, following the initial formulation of electroweak theory by Glashow, Weinberg, and Salam, before the work of 't Hooft and Veltman appeared, Salam discussed some interesting ideas about the renormalization of *non-polynomial scalar field theories* in his contribution to the 1970 Coral Gables Conference, summarizing work done in collaboration with Delbourgo, Koller, and Strathdee.

Very soon thereafter, this subject was largely swept aside by the tidal wave of research on non-Abelian gauge theories. More recently, however, the subject was revived in the context of PT symmetric field theories, as reviewed in Carl Bender's talk at Miami 2024.

During 2025 Galib Hoq and I revisited some of these ideas, from another point of view, as Galib will describe in detail in the following talk.

But perhaps some general remarks will serve as a useful segue into that work.

The idea is to consider scalar field theory with potentials $V(\phi)$ that involve arbitrary powers of ϕ. As a representative example, consider a Gaussian

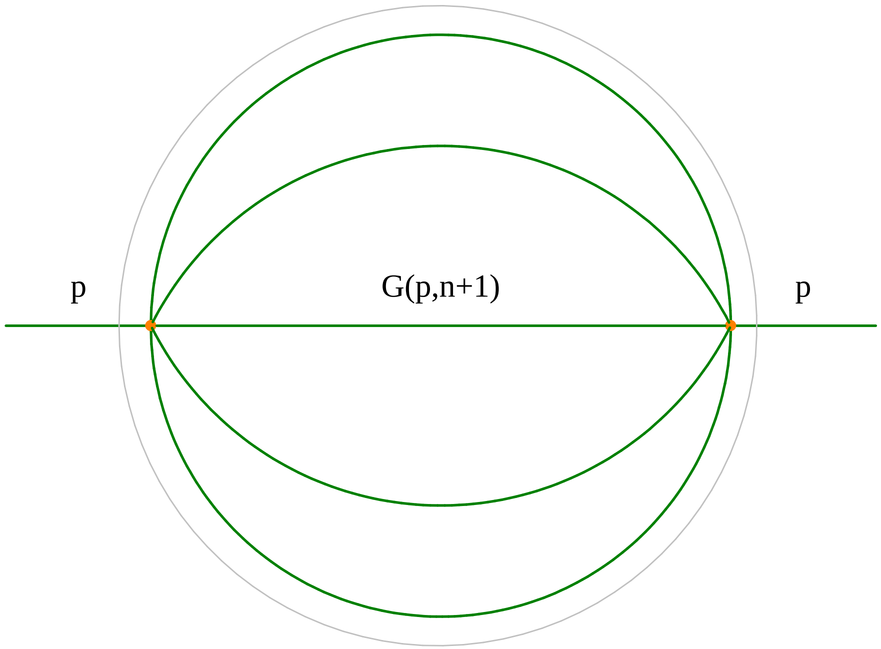
$$V(\phi) = \kappa \exp\left(-\lambda \phi^2\right)$$

Expanding the exponential leads to terms $\left(\phi^2\right)^n$ for all positive integer n, and therefore, as any student taking a standard course on QFT will recall, for $n > 2$ the resulting quantum perturbation theory in 4D spacetime has very bad high-energy behavior, an unacceptable infinite number of independently divergent Feynman diagrams, and correspondingly, the model is *not* renormalizable.

Ah, but an optimist might note there are two "coupling constants" κ ("major") and λ ("minor"), hence perturbation theory leads to a double expansion in powers $\kappa^k \lambda^\ell$. Could perhaps one, if not both, of the expansion sums be carried out, in closed-form, such that non-perturbative effects save the day? Well ... perhaps. Perhaps not.

Let's have a closer look at the sum over powers of λ for a fixed power of κ.

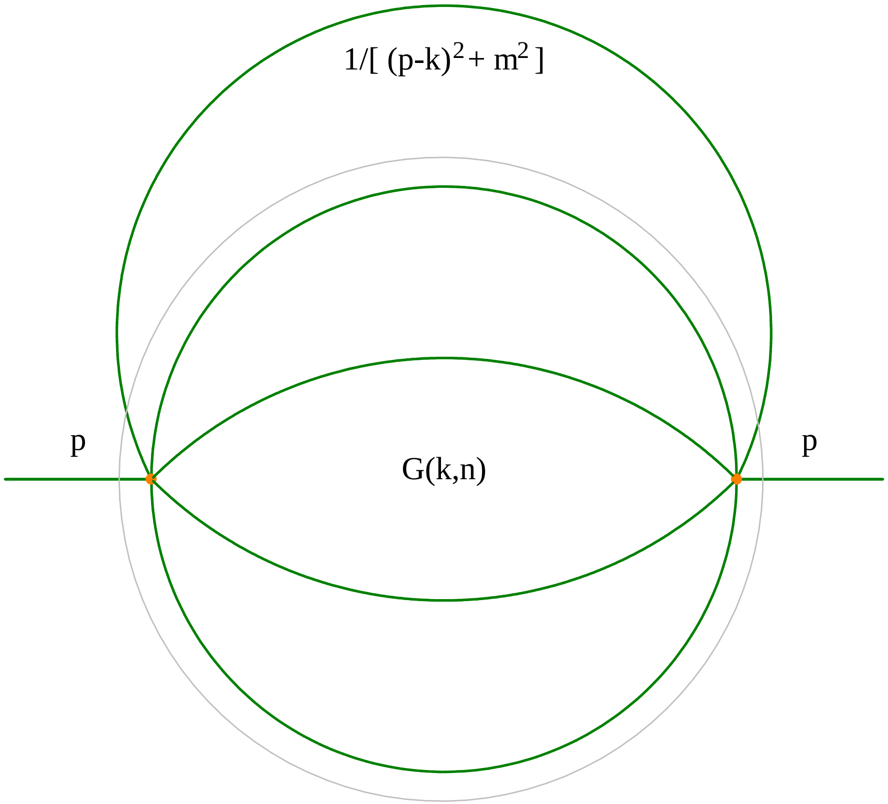
A typical $O(\kappa^2)$ process involves a sum over diagrams with several internal lines connecting two vertices, weighted by the appropriate power of λ and combinatoric factors.



A representative $O(\kappa^2)$ momentum-dependent correction, expressed as an n-loop diagram with $n+1$ massive propagators connecting the two vertices.

We call these “Hammock Diagrams” ... for obvious reasons.

Representing *the Leisure of the Theory Class*
[pun intended]



The same $O\left(\kappa^{2}\right)$ momentum-dependent correction, expressed as a single massive propagator convolved with an $(n-1)$-loop diagram with n massive propagators.

This leads to the recursion relation

$$G_{n+1}\left(p^2\right) = \int \frac{1}{\left(p-k\right)^2 + m^2}\; G_n\left(k\right) d^N k$$

starting with $G_1\left(p\right) = 1/\left(p^2 + m^2\right)$. Dimensional analysis gives

$$G_n\left(k^2\right) = \left(k^2\right)^{(n-1)N/2-n} g_n\left(\frac{m^2}{k^2}\right) \quad \text{with} \quad g_1\left(\frac{m^2}{k^2}\right) = \frac{k^2}{k^2+m^2}$$

So the recursion to be solved in Euclidean momentum space is

$$\left(p^2\right)^{nN/2-n-1} g_{n+1}\left(\frac{m^2}{p^2}\right) = \int \frac{\left(k^2\right)^{(n-1)N/2-n}}{\left(p-k\right)^2 + m^2}\; g_n\left(\frac{m^2}{k^2}\right) d^N k$$

For $m^2 \neq 0$ this is a challenging recursion relation.

But for massless propagators, $m^2 = 0$, using *dimensional regularization*, $g_n(0)$ is just an N-dependent number. The solution is then easily found to be

$$G_n\left(p^2\right) = \left(p^2\right)^{(n-1)N/2-n} \pi^{\frac{n-1}{2}N} \frac{\left(\Gamma\left(\frac{1}{2}(N-2)\right)\right)^n}{\Gamma\left(\frac{1}{2}n(N-2)\right)} \Gamma\left(n - \frac{1}{2}(n-1)N\right)$$

For any fixed integer $n \geq 2$ the *only* divergences in $G_n(p)$, as N approaches an integer larger than 2, are given explicitly by the final $\Gamma\left(n - \frac{1}{2}(n-1)N\right)$.

For example, if $N = 4 - 2\varepsilon$, $G_n\left(p^2\right)$ exhibits a simple $1/\varepsilon$ pole for all $n \geq 2$. The final Γ factor above gives

$$\Gamma\left(n - \frac{1}{2}(n-1)(4-2\varepsilon)\right) = \Gamma\left(2 - n + (n-1)\varepsilon\right) \underset{\varepsilon\to 0}{\sim} \frac{(-1)^n}{(n-1)!}\frac{1}{\varepsilon}$$

There are no $1/\varepsilon^2$ terms.[1] Hence the well-known 2-loop correction in conventional ϕ^4 theory, $G_3\left(p^2\right) \underset{\varepsilon\to 0}{\sim} \frac{1}{4}\pi^4 p^2\left(-\frac{1}{\varepsilon} + 2\ln p^2\right)$, which implies that model is *not* asymptotically free.

[1] This feature of $G_n\left(p^2\right)$ can be understood, *without* actual evaluation, by expressing the diagram as a dimensionally regularized dispersion relation.

Consider the $O(\kappa^2)$ propagator correction

$$\frac{1}{2}\int d^N u \int d^N v \;\; \langle 0| \;\; \phi(x) \;\; \exp\left(-\lambda\phi^2(u)\right) \;\; \exp\left(-\lambda\phi^2(v)\right) \;\; \phi(0) \;\; |0\rangle$$

When transformed to Euclidean (Wick rotated) momentum space the combinatorics lead to a self-energy as a *sum over powers of the minor coupling* λ of one-particle irreducible diagrams, as given by

$$2\sum_{n=2}^{\infty}\frac{(2n)!}{n!\,(n-1)!}\lambda^{2n}G_{2n-1}\left(p^2\right) = \frac{1}{\left(p^2\right)^{N-1}}\frac{2}{\pi^N\Gamma\left(\frac{1}{2}(N-2)\right)}\sum_{n=2}^{\infty}\left(w_N(p)\,\lambda^2\right)^n\frac{(2n)!}{n!\,(n-1)!}\frac{\Gamma\left(N-1-(N-2)\,n\right)}{\Gamma\left(\frac{1}{2}(2n-1)(N-2)\right)}$$

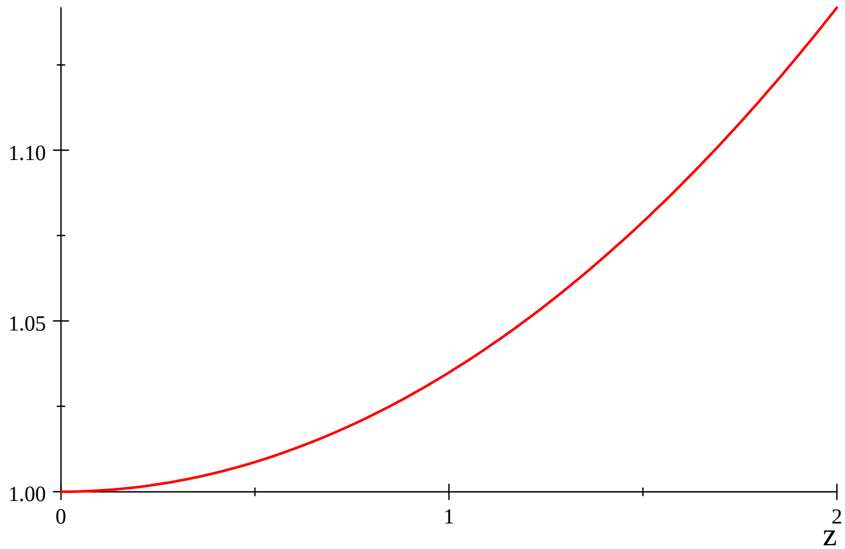
where $w_N(p) = \left(p^2\right)^{N-2}\pi^N\Gamma^2\left(\frac{1}{2}(N-2)\right) \underset{N\to 4}{\longrightarrow} \left(\pi^2 p^2\right)^2$. So for $N = 4-2\varepsilon$ we are left with a sum of simple $1/\varepsilon$ pole terms as $\varepsilon \to 0$. Again, there are no $1/\varepsilon^2$ terms. Summing the usual geometric series for self-energy insertions then leads to a modified "super propagator".

Keeping only the pole part of the diagrams, the self-energy term is

$$2\sum_{n=2}^{\infty}\frac{(2n)!}{n!\,(n-1)!}\,\lambda^{2n}G_{2n-1}\left(p^2\right)\underset{\varepsilon\to 0}{\sim}$$

$$-6\pi^4\underbrace{\lambda^4p^2\left(\frac{1}{\varepsilon}\right)}\left({}_1F_4\left([1],\left[2,2,2,\frac{3}{2}\right],\frac{1}{4}\lambda^2\pi^4p^4\right)+\frac{\lambda^2\pi^4p^4}{72}\,{}_1F_4\left([2],\left[3,3,3,\frac{5}{2}\right],\frac{1}{4}\lambda^2\pi^4p^4\right)\right)$$

just ϕ^4 theory multiplied by entire functions of p^4 that suggest *non-locality*



${}_1F_4\left([1],\left[2,2,2,\frac{3}{2}\right],\frac{1}{4}z^2\right)+\frac{z^2}{72}\,{}_1F_4\left([2],\left[3,3,3,\frac{5}{2}\right],\frac{1}{4}z^2\right)$ versus $z=\lambda\pi^2p^2$

So ... a *single* Gaussian field potential looks problematic, or at least not very promising.

But what happens when the field potential is a linear *superposition* of such Gaussians?

The next speaker will address that possibility.

Thank you for your time and attention.

And once again, welcome to

Extra Slides

Just for the sake of argument ... in homage to Kelly Stelle's work adding higher derivatives to the gravitational action.

Suppose we pump up the power of momentum in the propagator. Then the Wick-rotated recursion relation for hammock graphs becomes, e.g.,

$$G_{n+1}\left(p^2\right) = \int \frac{1}{(p-k)^4 + m^4} G_n\left(k\right) d^N k \tag{1}$$

where $G_1\left(p\right) = 1/\left(p^4 + m^4\right) = \left(1/\left(p^2 - im^2\right) - 1/\left(p^2 + im^2\right)\right)/\left(2im^2\right)$. (Complex poles and ghosts ... ah well, "Damn the torpedoes!" Etc.)

Dimensional analysis now gives

$$G_n\left(k^2\right) = \left(k^2\right)^{(n-1)N/2-2n} g_n\left(\frac{m^2}{k^2}\right) \quad \text{with} \quad g_1\left(\frac{m^2}{k^2}\right) = \frac{k^4}{k^4+m^4} \tag{2}$$

In the massless case we have the following recursion relation for $n > 1$ with initial condition $g_1 = 1$, as follows from $G_{n=1}(p) = 1/p^{4.}$.

$$\left(p^2\right)^{nN/2-2n-2} g_{n+1} = g_n \int \frac{1}{\left(k^2\right)^{2n+(1-n)N/2}\left((p-k)^2\right)^2}\, d^N k \tag{3}$$

where g_n is N-dependent. The resulting recursion relation for g_n is

$$g_{n+1} = \frac{\Gamma\left(\frac{1}{2}(N-2)\right)\Gamma\left(\frac{1}{2}(N-4)n+1\right)\Gamma\left(2-\frac{1}{2}(N-4)n\right)}{\Gamma\left(\frac{1}{2}(N-4)n+\frac{1}{2}N\right)\Gamma\left(\frac{1}{2}N-\frac{1}{2}(N-4)n\right)}\pi^{\frac{1}{2}N} g_n \tag{4}$$

Only now the limit as $N \to 4$ is *finite*. In that limit, $\lim_{N\to 4} \frac{\Gamma\left(\frac{1}{2}(N-2)\right)\Gamma\left(\frac{1}{2}(N-4)n+1\right)\Gamma\left(2-\frac{1}{2}(N-4)n\right)}{\Gamma\left(\frac{1}{2}(N-4)n+\frac{1}{2}N\right)\Gamma\left(\frac{1}{2}N-\frac{1}{2}(N-4)n\right)} = 1$, so

$$g_{n+1} = \pi^2 g_n \quad \text{for} \quad N = 4 \tag{5}$$

$$G_n\left(p^2\right) = \frac{1}{p^4}\left(\pi^2\right)^{n-1} \quad \text{for} \quad N = 4 \tag{6}$$

In Lorentzian momentum space, for massless fields (sans "lobes") subject to a Gaussian field potential sans constant and mass term, $V(\phi) = \kappa\left(\exp\left(-\lambda\phi^2\right) - 1 + \lambda\phi^2\right) = \frac{1}{2}\kappa\lambda^2\phi^4 + O\left(\phi^6\right)$, the $O\left(\kappa^2\right)$ correction to the propagator would be

$$\kappa^2 \sum_{\text{even } n=2}^{\infty} \lambda^{2n}\frac{(2n)!}{(n!)^2}(2n)\, G_{n-1}\left(-p^2\right) = \frac{\kappa^2}{\pi^4 p^4}\pi^2\lambda^2\left(\frac{1}{\left(1-4\pi^2\lambda^2\right)^{3/2}} - \frac{1}{\left(1+4\pi^2\lambda^2\right)^{3/2}}\right) \tag{7}$$

upon using a bit of help from Mathematica: $\sum_{k=1}^{\infty} \frac{(4k)!}{(2k)!(2k-1)!}(\pi\lambda)^{4k} = \pi^2\lambda^2\left(\frac{1}{\left(1-4\pi^2\lambda^2\right)^{3/2}} - \frac{1}{\left(1+4\pi^2\lambda^2\right)^{3/2}}\right)$.

Summing the usual geometric series then leads to the modified "super propagator" to this order:

$$\Delta(p) = \frac{p^4}{p^8 - \frac{\kappa^2\lambda^2}{\pi^2}\left(\frac{1}{\left(1-4\pi^2\lambda^2\right)^{3/2}} - \frac{1}{\left(1+4\pi^2\lambda^2\right)^{3/2}}\right)} \tag{8}$$

which, unless I've trashed a sign, has a "Landau pole" or "ghost".

Ah, but here the theory is *finite* to every order in κ. So one cannot just jump to the conclusion that $\kappa = 0$ when the cut-off is removed. There is no cut-off! Although there is indeed a ghost in the machine ... as evinced by the $1/p^4$ propagator from the get-go. Note that

$$\frac{p^4}{p^8 - m^8} = \frac{1}{4m^2\left(p^2 - m^2\right)} - \frac{1}{4m^2\left(p^2 + m^2\right)} + \frac{1}{2\left(p^4 + m^4\right)} \tag{9}$$

It not only has a ghost, it has a *tachyonic* ghost. Food for thought?