PHYSICS COLLOQUIUM

"Quantizing the Hendrik Antoon Lorentz force, 1853 - 1928, University of Leyden, Holland"

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Synopsis. The Maxwell equations are not complete, unless supplemented by the Lorentz-force equation that gives the force on the elementary charges in a medium. The first consistent co-variant formulation was given by H.A. Lorentz in 1902 (Proc. Roy. Acad. of Arts and Sciences, Amsterdam, 254, 1902; Dutch: VKAW 254, 1902). With K^{α} being the Minkowski four-vector force, we have

$$K^{\alpha} \equiv \tfrac{dp^{\alpha}}{d\tau} = m\tfrac{dU^{\alpha}}{d\tau} = \tfrac{\mathbf{q}}{c} F^{\alpha\beta} U_{\beta} \,. \tag{1}$$

For static fields, the well-known non-relativistic result reads,

$$\mathbf{F} = \textstyle\sum_i q_i \left[\mathbf{E} + (\mathbf{v}_i / c) \times \mathbf{B}\right], \tag{2}$$

whereby we note that the units are 'rationalized Gaussian' or Heaviside–Lorentz. Herein the v_i are the real particle velocities, *not* relative to the aether, which was discarded after Michelson–Morley (1887). In the semi-quantal description of band electrons in solids, the velocities are the expectation values in Bloch states of energy $\mathsf{E}(\mathbf{k})$ with

$$\mathbf{v_k} \equiv \langle \mathbf{k} \,|\, \mathbf{v} \,|\, \mathbf{k} \rangle = \hbar^{-1} \nabla_{\mathbf{k}} \mathsf{E}(\mathbf{k}). \tag{3}$$

Note that the off-diagonal matrix elements for Bloch states are zero: $\langle \mathbf{k} \,|\, \mathbf{v} \,|\, \mathbf{k}' \rangle = \mathbf{v_k} \delta_{\mathbf{kk}'}$.

Unfortunately, there is no k-space in the plane $\perp \mathbf{B}$. Let the criterion be $\omega_c \tau_c \sim 0.1$ (i.e. 10% of the orbit is completed prior to a collision occurring), where $\omega_c = eB / m^* c$ is the cyclotron frequency and τ_c is the collision time, then one finds that B should not exceed 100 Gauss or 10^{-2} Tesla. *We thus need a full quantum theory for inductions larger than a mere* 0.01 T!

In the first part of this talk we shall ignore the above and briefly dwell on standard transport theory based on the Boltzmann Transport Equation, solving the latter to first order Enskog approximation, assuming quasi-elastic collisions. The non-dissipative magnetic force is placed at the rhs of the BTE, where it only 'curves' the path between collisions. If the energy surfaces in k-space are spheres, the effect of a magnetic field is the 'mere' replacement of the scalar collision time by an effective collision tensor $\boldsymbol{\tau}$, which depends on the cyclotron frequency. All galvanomagnetic phenomena, as well as thermomagnetic phenomena can be described this way. A few simple examples show that the results are sometimes close to being correct, while in other cases the results are clearly entirely off. Results of *dozens* of well-known books (e.g., Ziman Electrons and Phonons, Oxford 1960) which "remedy" the situation by using special band struc-

tures, etc., [even up into the present century!] must be firmly *discarded*!

Quantum mechanics was invented to be used! Thus in the second part of this talk, we look at the solutions of the appropriate Schrödinger equation for the Hamiltonian

$$\mathbf{H}^0 = [\mathbf{p} - (\mathbf{q}/c)\mathbf{A}]^2 / 2m^* + U(z)\,, \tag{4}$$

which applies in 3D,2D or 1D space, with the magnetic field being in the z-direction. The most often employed Landau gauge, $\mathbf{A} = (0, Bx, 0)$, leads to harmonic oscillator eigenfunctions. The new results cannot be used in conjunction with the Boltzmann equation, however, but are eminently suitable for use with results based on LRT (linear response theory) providing randomization is carried out on the many-body level. Surprisingly, while perturbations to all orders are very complex, the closed-form results obtained from it by myself and coworkers are manageable and tailor-cut for applications! When applied to the ordinary 3D Hall effect, the pertinent point is that only non-diagonal velocity operator matrix elements exist. Our non-diagonal conductance formulae then lead to the standard result for any degenerate or non-degenerate electron/hole gas, with only the factor $(3\pi/8)$ missing for the latter. Thus, one may ask: So what?

The crown of the work is that exactly the same formalism in two dimensions gives *nothing akin* to the classical results, but out ‘tumbles’ without any fanfare the renowned integer Quantum Hall effect (QHE), discovered by Nobel Laureate Von Klitzing *cum suis*(1980), while our formalism was being published! Needless to say that all oscillatory galvanomagnetic and thermomagnetic phenomena, as well as novel nano-confinement effects, can be similarly treated – with enough perseverance! [Formulas will be limited to what is essential and intelligent only.]

Wilder Auditorium

J.L. Knight Physics Building Room 112

Wednesday, February 23rd, 5:00 pm

Refreshments will be served at 4:45 p.m. in the Physics Library