Hom-algebra structures

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Abstract

In this colloquium lecture an introductory overview on the subject of Hom-algebra structures will be given with emphasize on hom-algebra generalizations of Lie algebras and associative algebras.

In 1990s several pioneering on quantum deformations of algebras, q-deformed oscillator algebras, q-deformations of Witt and Virasoro algebras and related families of algebras defined by generators and parameter commutation relations, were constructed in connection to quantum deformations and discretized models of mechanics and quantum mechanics, q-deformations of vertex operators, q-deformed conformal quantum field theory, q-deformed integrable systems, q-deformed superstrings and central extensions. Also various quantum n-ary extensions of Numbu mechanics and related n-ary extensions of differential structures and of Lie algebras Jacobi identities were considered. It was noticed in particular that many of quantum algebras and q-deformed Lie algebras obey certain q-deformed versions of Jacobi identity generalizing Lie algebras Jacobi identity. Motivated by these works, I and my collaborators, Hartwig, and Larsson, developed in 2003 a general method of obtaining such deformations and generalized Jacobi identities based on general twisted derivations. This development, as well supersymmetric generalizations, lead to development of more general algebraic structures such as quasi-Lie and Hom-Lie algebras, Hom-associative and Hom-Lie admissible algebras, Hom-Jordan algebras, Hom-Poisson algebras, Hom-Yang-Baxter equations, Hom-bialgebras, Hom-Hopf algebras, and other hom-algebra structures, as well as Hom-Nambu and Hom-Nambu Lie algebras some related n-ary Hom-algebra generalizations of Nambu algebras, associative algebras and Lie algebras and their constructions.

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